# Maths for Physics 

## Basic Skills Booklet

September 2024

This booklet belongs to

## Maths for Physics - Basic skills

The purpose of this booklet is to act as a reference for the basic maths skills that you must have in order to be successful on the A Level Physics course. All of the topics, apart from the prefixes used in Physics, should have been covered on your GCSE maths course.

However, you must be absolutely confident using ALL of the following key skills.
The key skills are:

1. Expressing numbers in Standard Form.
2. Expressing numbers to a specified number of significant figures.
3. Calculating the percentage change in values.
4. Performing calculations on a calculator with numbers in standard form.
5. Recalling and using Physics prefixes.
6. Converting between units
7. Simple trigonometry
8. Re-arranging formulae.
9. Performing complex calculations on a calculator.
10. Answering a question (putting all the skills together).

The answers to the example questions in each section can be found in:
11. answers to example questions

## 1. Expressing numbers in standard form.

Standard form is very useful when expressing numbers that are either very big or very small.

The number " 320,000 " is the same as " $3.2 \times 100,000$ " $(3.2 \times 100,000=320,000)$

As $10^{5}=100,000, ~ " 3.2 \times 100,000$ " can also be expressed as " $3.2 \times 10^{5 "}$. This is a far more convenient way of writing 320,000 .

So,

$$
320,000=3.2 \times 10^{5}
$$

Similarly, $\quad 3.713 \times 10^{2}=371.3$.

The number 0.000032 can be represented as " $3.2 \div 100,000$ ". This can also be written as:

$$
0.000032=\frac{3.2}{100,000}=\frac{3.2}{10^{5}}=3.2 \times \frac{1}{10^{5}}=3.2 \times 10^{-5}
$$

Again, $3.2 \times 10^{-5}$ is a more convenient way of writing 0.00032 .

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So, $0.000032=3.2 \times 10^{-5}$

Similarly, $\quad 3.713 \times 10^{-2}=0.003713$
Convert the following numbers to and from standard form.
TABLE 1

|  | Value | What is it? | Expressed in <br> standard form |
| :---: | :---: | :---: | :---: |
| 1 | $9,460,528,400,000,000 \mathrm{~m}$ | a light year | $9.4605284 \times 10^{15} \mathrm{~m}$ |
| 2 | $101,325 \mathrm{~Pa}$ | Pa |  |
| 3 | m | atmospheric <br> pressure | Moon-Earth <br> separation |
| 4 | 0.02 m | $3.84403 \times 10^{8} \mathrm{~m}$ |  |
| 5 | speed of light | $2.9979 \times 10^{8} \mathrm{~ms}^{-1}$ |  |
| 6 | sper <br> molecules per mole | Avogadro's <br> number | m |
| 7 | 0.000000001 m | microwave <br> wavelength | $2 \times 10^{-2} \mathrm{~m}$ |
| 8 | 0.000000515 m | diameter of <br> an atom | m |
| 9 | wavelength <br> of visible <br> light | resistivity of <br> copper | $1.72 \times 10^{-8} \Omega \mathrm{~m}$ |

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2. Expressing numbers to a specified number of significant figures Answers to calculations in Physics are also expressed to a specific number of significant figures. Unless specifically asked for in a question, ALL of your answers should ALWAYS be given in a decimal format to 3 significant figures.

Answers should:

- always be expressed as a decimal number, this means:
- NO FRACTIONS eg $-\frac{1}{3}$ should be written as -0.333
- NO MULTIPLES of $\pi$, eg $2 \pi$ should be written as 6.28
- never be RECURRING numbers, so don't use the "damned spot" eg $3 . \dot{3}$

An answer of 3.33333 recurring, should be written as 3.33 (to 3 significant figures).

Complete the following tables, expressing the numbers to the appropriate number of significant figures. Remember to round the "right most" significant figure.

TABLE 2

| Number | Number of significant figures |  |  |
| :---: | :---: | :---: | :---: |
|  | 3 | 2 | 1 |
| 3.14235 | 3.14 | 3.1 | 3 |
| 3.009442 |  |  |  |
| 45.336 |  |  |  |
| 72.49513 |  |  |  |
| 505.334 |  |  |  |
| 793.4591 |  |  |  |
| $3,045.778$ |  |  |  |
| $5,139.113$ |  |  |  |
| $76,493.2$ |  |  |  |
| $43,395.8$ |  |  |  |
| $3.9751 \times 10^{4}$ |  |  |  |
| $44.964 \times 10^{3}$ |  |  |  |

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TABLE 3

| Number | Number of significant figures |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{1}$ |
| 0.93552 | 0.936 | 0.94 | 0.9 |
| 0.95387 |  |  |  |
| 0.032532 |  |  |  |
| 0.0403912 |  |  |  |
| 0.0038461 |  |  |  |
| 0.00093571 |  |  |  |
| $4.8835 \times 10^{-3}$ |  |  |  |
| $34.9883 \times 10^{-5}$ |  |  |  |

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## 3. Calculating the percentage change in values

Some questions give the percentage change in a value, so you need to be able to calculate the actual amount that has changed using this information.

For example, if the price of a $£ 4.40$ coffee increases by $2.5 \%$,

$$
\text { Increase of } 2.5 \%: \frac{2.5}{100} \times 4.40=0.11
$$

So, final cost is $4.40+0.11=4.51$
If the price of coffee decreased by $2.5 \%$, then
the final cost is $4.40-0.11=4.29$

An alternative to how these percentage changes can be written is as follows:
The mass of a block is reduced by $5.0 \%$ to 4.0 kg . What was the initial mass of the block?

Suppose $Y$ was the original mass of the block, then, write out the change algebraically,

$$
\begin{aligned}
& Y-(5 \% \text { of } Y)=4.00 \\
& Y-\left(\frac{5}{100} \times Y\right)=4.00 \\
& Y-\frac{5}{100} Y=4.00 \\
& Y\left(1-\frac{5}{100}\right)=4.00 \\
& Y\left(\frac{95}{100}\right)=4.00 \\
& Y=\frac{100}{95} \times 4.0=4.21 \mathrm{~kg}
\end{aligned}
$$

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Calculate, and fill in, the missing values in the table below.
Give all your answers to 3 significant figures

TABLE 4

| Initial value | Change | \% | Final value |
| :---: | :---: | :---: | :---: |
| 2.55 | Increases by | 1.70 | 2.59 |
| 32.2 | Increases by | 28.4 |  |
| 892 | Decreases by | 35.2 |  |
|  | Increases by | 0.331 | 90.6 |
| 73.8 | Decreases by | 8.95 |  |
| 51.6 | Decreases by | 10.4 | 0.737 |
| 274 | Increases by |  | 53.3 |
|  | Decreases by |  | 255 |

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4. Performing calculations on a calculator with numbers in standard form

Some students have been taught to enter a number in standard form, for example $3.2 \times 10^{5}$, into their calculator in the following way:


Unfortunately, the calculator treats the 3.2 and $10^{5}$ as separate things which can lead to all sorts of problems in calculations.

You MUST instead use the $\times 10^{x}$ key (or $\exp$ key on older calculators) when entering numbers in standard form. The diagram below shows where these keys can be found on a Casio calculator, together with how the numbers $3.2 \times 10^{5}$ and $3.2 \times 10^{-5}$ should be entered.


Entering the numbers in this way makes sure the calculator handles the numbers correctly.

So, the following calculation: $3.2 \times 10^{5} \times 9.9 \times 10^{-3}$ would be entered as:


The answer should be 3168 , which expressed to 3 sig fig (significant figures) would be 3170 .

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Complete the calculations in the table below, expressing your answers to 3 significant figures.

## TABLE 5

| Calculation | Answer |
| :---: | :---: |
| $3.11 \times 10^{4} \times 7.85 \times 10^{3}$ |  |
| $45.22 \times 10^{-3} \times 1.077 \times 10^{-6}$ |  |
| $435 \times 10^{-9} \div 58.8 \times 10^{6}$ |  |
| $0.55 \times 10^{5} \div 2.22 \times 10^{-5}$ |  |
| $\left(17.3 \times 10^{2}\right)^{2}$ |  |
| $3.33 \times 10^{-12} \times\left(4.053 \times 10^{6}\right)^{2}$ |  |
| $173 \times 10^{-9} \div\left(4.44 \times 10^{3}\right)^{3}$ |  |

## 5. Prefixes used in Physics

Make sure you are clear on the difference between units (of measurement), symbols and quantities as they do different jobs.

## Examples:

The quantity "Time" is measured in units of seconds. The units can be abbreviated to 's'. In an equation that needs you to multiply something by 'time', time will be represented by the symbol ' $t$ '.

The quantity "Mass" is measured in units of kilograms. The units can be abbreviated to ' kg '. In an equation that needs you to multiply by mass, then it is given the symbol ' $m$ '

In the famous equation $E=m c^{2}$ :
The symbol 'E' represents the quantity "energy" - it would have units of Joules (J) The symbol ' $m$ ' represents the quantity "mass" - it would have units of kilograms (kg)
The symbol 'c' represents the quantity "speed of light" - it would have units of $\mathrm{ms}^{-1}$ (this is metres per second)

Often in Physics, units are written using so called prefixes which is an even shorter way of writing numbers than standard form.

For example instead of writing $2.95 \times 10^{-9} \mathrm{~m}$ we can write 2.95 nm where n means nano and is a short way of writing $\times 10^{-9}$.

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Here is a table that shows all the prefixes you may come across in A level Physics. Some crop up more frequently than others, but you need to know them all.

| Prefix | Symbol | Multiplier |
| :---: | :---: | :---: |
| atto | $a$ | $10^{-18}$ |
| femto | $f$ | $10^{-15}$ |
| pico | $p$ | $10^{-12}$ |
| nano | $n$ | $10^{-9}$ |
| micro | $\mu$ | $10^{-6}$ |
| milli | $m$ | $10^{-3}$ |
| centi | $c$ | $10^{-2}$ |
| kilo | $k$ | $10^{3}$ |
| mega | $M$ | $10^{6}$ |
| giga | $G$ | $10^{9}$ |
| tera | $T$ | $10^{12}$ |

Before performing a calculation, the values you are given must be expressed in their so called base units (some of which are known as SI units). Of the 7 SI units, you will only come across 6 on the Physics course, they are:

| Quantity | Base (SI) Unit |  |
| :--- | :---: | :---: |
| Mass, m | kg | kilogram |
| Length, L | m | metre |
| Time, $\dagger$ | s | second |
| Electric current, I | A | ampere |
| Temperature, T | K | kelvin |
| Amount of substance, n | mol | mole |

In fact, every unit (eg Watts, Joules) can be expressed in terms of these base units. NOTICE that MASS is expressed in KILOGRAM's, kg, NOT grammes, $g$.

Values given in questions will often be given with a prefix, so you must be able to convert this into a number to the power of ten so that you can carry out the correct calculation on a calculator.

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Note that as this is only to allow you to use the numbers in calculations, you do NOT need to express the number in standard form. The prefix is simply replaced by the appropriate power of 10 , for example

320 nm can be expressed as $320 \times 10^{-9} \mathrm{~m}$ (as nano, $n$, means $\times 10^{-9}$ ), you DO NOT need to express this as $3.2 \times 10^{-7} \mathrm{~m}$.

Express the following values in their base units without the prefix:

## TABLE 6

| Prefix value | In SI Units |
| :---: | ---: |
| 1 GPa | Pa |
| 1.75 kJ | J |
| 0.35 MHz | Hz |
| 32.0 mC | C |
| 640 km | m |
| 52 kg |  |
| 450 nm |  |
| $0.73 \mathrm{\mu s}$ |  |
| 43.9 TV |  |
| 470 pF |  |

Sometimes questions will ask for an answer to be expressed with a particular prefix. You also need to be express base unit values with an appropriate prefix.

Fortunately, Casio calculators have a button to help do this; it's the ENG button which is short for Engineering. Pressing this button will express a number in one of the prefix power of tens. Do the following calculation.
$32 \times 10^{2} \times 13 \times 10^{3}$, you should find it equals $41,600,000$.
Now press the ENG key, you should see that the number is expressed as $41.6 \times 10^{6}$. If you press the ENG key again, it will show $41600 \times 10^{3}$.
If you now press ${ }^{\text {SHIFT }}$ then ENG you will find that the display shows $41.6 \times 10^{6}$.

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Express the following base unit values with an appropriate prefix

## TABLE 7

| Base unit value | Prefix value |
| :---: | :---: |
| $3.2 \times 10^{9} \mathrm{~m}$ |  |
| $0.72 \times 10^{5} \mathrm{~N}$ |  |
| $1.27 \times 10^{4} \mathrm{~Hz}$ |  |
| $3.2 \times 10^{-7} \mathrm{~s}$ |  |
| $2.35 \times 10^{-8} \mathrm{~m}$ |  |
| $48.3 \times 10^{7} \mathrm{~W}$ |  |
| $0.36 \times 10^{-11} \mathrm{~F}$ |  |
| $0.945 \times 10^{14} \mathrm{~V}$ |  |

## 6. Converting between units.

Sometimes values are given for areas in one unit and you need to be able to convert them into another unit, for example:

Convert $12.3 \mathrm{~mm}^{2}$ into $\mathrm{m}^{2}$.

This conversion is not the same as converting 12.3 mm into m (which from the previous section would simply involve replacing the prefix milli, $m$, with the appropriate power of 10 , ie $10^{-3}$, hence $12.3 \mathrm{~mm}=12.3 \times 10^{-3} \mathrm{~m}$ ).

With $\mathrm{mm}^{2}$ however, the conversion factor $\left(10^{-3}\right)$ must also be squared. This means that $12.3 \mathrm{~mm}^{2}=12.3 \times\left(10^{-3}\right)^{2} \mathrm{~m}^{2}=12.3 \times 10^{-6} \mathrm{~m}^{2}$.

The same rules apply for converting $\mathrm{mm}^{3}$ to $\mathrm{m}^{3}$. In this case the conversion factor must be cubed.

So converting $12.3 \mathrm{~mm}^{3}$ into $\mathrm{m}^{3}$ would be given by:
$12.3 \mathrm{~mm}^{3}=12.3 \times\left(10^{-3}\right)^{3} \mathrm{~m}^{3}=12.3 \times 10^{-9} \mathrm{~m}^{3}$

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Convert the given values into the unit shown. The first line has been completed as an example.

## TABLE 8

| Convert the following | Into this unit | Answer |
| :---: | :---: | :---: |
| $3.94 \mathrm{~mm}^{2}$ | $\mathrm{~m}^{2}$ | $3.94 \times 10^{-6}$ |
| 6.71 cm | m |  |
| $0.883 \mu \mathrm{~m}^{2}$ | $\mathrm{~m}^{2}$ |  |
| $78.3 \mathrm{~mm}^{3}$ | $\mathrm{~m}^{3}$ |  |
| $68.5 \mathrm{~cm}^{2}$ | $\mathrm{~m}^{2}$ |  |
| $3.81 \mu \mathrm{~m}$ | $\mathrm{~m}^{3}$ |  |
| $28.8 \mathrm{~cm}^{3}$ | $\mathrm{~mm}^{2}$ |  |
| $7.31 \mu \mathrm{~m}^{2}$ |  |  |

## Angles

As well as units, you also need to be able to convert between degrees, ${ }^{0}$, and radians, rad. Provided you remember the relationship that $360^{\circ}$ is equal to $2 \pi \mathrm{rad}$, angles given in one unit can be easily converted into the other.

Write down the number of degrees and radians in a complete circle as shown:

$$
\overline{360}=\frac{\overline{2 \pi}}{}
$$

Then add in the value you have on the appropriate side of the equation:

$$
\text { angle in degrees } \longrightarrow \frac{}{360}=\frac{}{2 \pi} \longleftrightarrow \text { angle in radians }
$$

For example, convert $90^{\circ}$ into radians.

$$
\text { angle in degrees } \longrightarrow \frac{90}{360}=\frac{}{2 \pi}
$$

Re-arranging this gives:

$$
\begin{aligned}
& \frac{90}{360} \times 2 \pi= \\
& \quad=\frac{\pi}{2}=1.57 \mathrm{rad}
\end{aligned}
$$

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Although in Maths answers can be expressed, conveniently, in terms of $\pi$, answers in Physics are always expressed as a decimal to a set number of significant figures (typically 3). Answers in Physics should therefore ALWAYS be given as a decimal answer and not left as a multiple of $\pi$.

Complete the table with the missing values. Give your answers to 3 significant figures.

## TABLE 9

| Angle in degrees | Angle in radians |
| :---: | :---: |
| 180 | 3.14 |
|  | 0.785 |
| 73.1 | 9.42 |
| 1.57 |  |
| 18.0 |  |
|  | 3.14 |
|  | 4.71 |

## 7. Simple trigonometry.

All of the trigonometry will be restricted to right angle triangles. This means sine, cosine and tangent (sohcahtoa) and Pythagoras' theorem must be remembered. The sine and cosine rules are not required.

For a right angle triangle, as shown below, " $a$ " is adjacent to the angle $\theta$, " 0 " is opposite the angle $\theta$ and $h$ is the hypotenuse.

a
You will need to learn the relationships between sine, cosine and tangent for a right angle triangle.

$$
\sin \theta=\frac{o}{h} \quad \cos \theta=\frac{a}{h} \quad \tan \theta=\frac{o}{a}
$$

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You will also need to learn and use Pythagoras' theorem:

$$
h^{2}=0^{2}+a^{2}
$$

Find the missing values in the following triangles
1.

2.

3.

$x=$ $\qquad$
$x=$ $\qquad$
$\qquad$
$y=$ $\qquad$ $\theta=$ $\qquad$
4.

5.


$$
x=\text {....................... }
$$

$\qquad$
$x=$
6.
4.1


[^0]
## Maths for Physics - Basic skills

## 8. Re-arranging formulae

In GCSE Physics the equations you come across are relatively straightforward to re-arrange so that answers can be calculated. With A Level Physics the equations are generally more complex, however, the approach to re-arranging them is the same.

It is ESSENTIAL that you are confident re-arranging formulae.

| Equation | Make this <br> the subject | Operation <br> (to both <br> sides) | Steps |
| :--- | :---: | :---: | :--- |
| $a=b+c$ | $-c]$ | $a-c=b+c-c$ <br> $a-c=b$ <br> or <br> $b=a-c$ |  |
| $a=b-c$ | $c=$ | $+c]$ | $a+c=b-c+c$ <br> $a+c=b$ <br> $a-a+c=b-a$ <br> $c=b-a$ |


| Equation | Make this the subject | Operation (to both sides) | Steps |
| :---: | :---: | :---: | :---: |
| $a=\frac{b}{c}$ | $b=$ | x c] | $\begin{aligned} & a \times c=\frac{b}{c} \times c \\ & a \times c=b \end{aligned}$ |
|  | $c=$ | $x c]$ $\div a]$ | $\begin{aligned} & a \times c=\frac{b}{c} \times c \\ & a \times c=b \\ & a \times c \div a=b \div a \\ & c=\frac{b}{a} \end{aligned}$ |
| $a=b c$ | $b=$ | $\div c]$ | $\begin{aligned} & a \div c=b c \div c \\ & \frac{a}{c}=b \end{aligned}$ |

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| $a=b c+d$ | $b=$ | $-\mathrm{d}]$ $\div c]$ | $\begin{aligned} & a-d=b c+d-d \\ & a-d=b c \\ & (a-d) \div c=b c \div c \\ & \frac{a-d}{c}=b \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $a=\frac{b}{c}-d$ | $c=$ | $+d]$ $x c]$ $\div(a+d)]$ | $\begin{aligned} & a+d=\frac{b}{c}-d+d \\ & a+d=\frac{b}{c} \\ & (a+d) \times c=\frac{b}{c} \times c \\ & c(a+d)=b \\ & c(a+d) \div(a+d)=b \div(a+d) \\ & c=\frac{b}{a+d} \end{aligned}$ |


| Equation | Make this the subject | Operation (to both sides) | Steps |
| :---: | :---: | :---: | :---: |
| $a=b c^{2}$ <br> or | $b=$ | $\left.\div c^{2}\right]$ | $\begin{aligned} & a \div c^{2}=b \times c^{2} \div c^{2} \\ & \frac{a}{c^{2}}=b \end{aligned}$ |
| $a=b \times c^{2}$ | $c=$ | $\div b]$ <br> • $]$ | $\begin{aligned} & a \div b=b \times c^{2} \div b \\ & \frac{a}{b}=c^{2} \\ & \sqrt{\frac{a}{b}}=\sqrt{c^{2}} \\ & c=\sqrt{\frac{a}{b}} \end{aligned}$ |
| $a=1 / 2 b c^{2}$ | $b=$ | $\times 2]$ $\left.\div c^{2}\right]$ | $\begin{aligned} & a \times 2=1 / 2 b \times c^{2} \times 2 \\ & 2 a=b \times c^{2} \\ & 2 a \div c^{2}=b \times c^{2} \div c^{2} \\ & \frac{2 a}{c^{2}}=b \end{aligned}$ |

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Re-arrange the following formula, making the letter in the brackets the subject of the equation.

TABLE 10
1.
(Q) $\quad \varepsilon=E / Q$
8.
(f) $h f=\phi+E_{m}$
2. $(\Delta \mathrm{Q}) \quad \mathrm{I}=\Delta \mathrm{Q} / \Delta \mathrm{t}$
9. (v) $E_{k}=1 / 2 m v^{2}$
3. (V) $\rho=m / V$
10. ( $\Delta \mathrm{L}) \quad \mathrm{E}=1 / 2 \mathrm{k} \Delta \mathrm{L}^{2}$
4.
(F) $E=1 / 2 F \Delta L$
11. (r) $\varepsilon=\mathrm{IR}+\mathrm{Ir}$
5.
(s) $\quad w=\lambda D / s$
12. (I)
$\varepsilon=\mathrm{IR}+\mathrm{Ir}$
6. $\left(E_{2}\right) \quad h f=E_{1}-E_{2}$ 13. (a) $v^{2}=u^{2}+2 a s$
7. ( $L$ ) $E=f L / A \Delta L$ 14. (a) $s=u t+1 / 2 a t^{2}$

## Maths for Physics - Basic skills

## 9. Performing complex calculations on a calculator

You need to be very careful when carrying out complex calculations on a calculator because of the way the calculator works. For example, calculate the following on your calculator

$$
\frac{30}{5 \times 3}=\text { ? }
$$

You should find that the answer is 2 , although you may have got 18! The calculator works things out in steps, so if you enter $30 \div 5 \times 3$ the calculator will work out each bit in turn, ie $30 \div 5=6 ; 6 \times 3=18$.

To calculate this correctly, you need to add brackets, $30 \div(5 \times 3)=2$.

Often, calculations in Physics involve multiplying many different numbers together. In these situations, the best way to avoid making a simple mistake when completing the calculation is to do the calculation in stages, rather than relying on putting in the correct brackets.

For example, to work out the following calculation:

$$
=\frac{\pi \times 1.25^{2}}{5.22 \times 10^{3} \times 9.00 \times 10^{-6}}
$$

Calculate the top and bottom parts separately, and write down the answers on a new line;

$$
=\frac{4.9087}{0.04698}
$$

THEN do the division;

$$
=104.48=104 \text { (to } 3 \text { sig fig!) }
$$

Use your knowledge of prefixes to express these numbers in their base units then your calculator to work out the answers to the following questions.

1. $x=32 \mathrm{~nm} ; y=6.602 \times 10^{15} \mathrm{Js} ; z=0.55 \mu \mathrm{~m}$.

Calculate the value of $\frac{x^{2} y}{z}$ giving your answer to 4 significant figures.
2. $a=101 \mathrm{kPa} ; b=78.3 \mathrm{GW} ; c=0.035 \mathrm{pF} ; \mathrm{d}=0.056 \mathrm{~J}$

Calculate the value of $(\mathrm{bc})+\frac{\mathrm{a}}{\mathrm{d}^{2}}$ giving your answer to 2 significant figures.

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## 10. Answering a question (putting all the skills together)

Calculate the velocity $(\mathrm{v})$ of a lorry with a mass of $1.1 \times 10^{3} \mathrm{~kg}$ that has a kinetic energy $E_{k}$ of 250 kJ .

Step 1. Figure out which equation you are going to use, in this case it is $E_{k}=1 / 2 \mathrm{mv}^{2}$.
Step 2. Write the equation down.
Step 3. Rearrange the equation to make $v$ the subject of the equation; showing each of your re-arrangement steps (you will often get marks for doing this correctly).

Step 4. Convert the values given in the question into the relevant base units.

Step 5. Write down the re-arranged equation with the VALUES rather than the letters.

Step 6. Calculate the answer.

Step 7. Sense check your answer - does it seem reasonable? (If not, check you haven't made a simple mistake).

Step 8. Write down the answer, to 3 significant figures.
Step 9. Write down the correct units.
So, this is what should be written on your paper:
$E_{k}=1 / 2 m v^{2}$
$2 E_{k}=m v^{2}$
$v^{2}=2 E_{k} / m$
$v=\sqrt{ }\left(2 E_{k} / m\right) ;$
$E k=250 \mathrm{~kJ}=250 \times 10^{3} \mathrm{~J}$
$v=\sqrt{ }\left(2 \times 250 \times 10^{3} / 1.1 \times 10^{3}\right)$
$v=\sqrt{ }\left(500 \times 10^{3} / 1.1 \times 10^{3}\right)$
$v=\sqrt{ } 454.5$
$v=21.3 \mathrm{~ms}^{-1}$.

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Rearrangement of equations-practice questions
a) Re-arrange the following equations, making the letter shown the subject:
b) Substitute the values, converted into base units, and calculate the value of the "new" subject.
c) NOTE: units of acceleration (" $a$ " below) are $\mathrm{ms}^{-2}$ and velocity ("u" or " $v$ " below) are $\mathrm{ms}^{-1}$. In both cases $m$ means metres, not milli.

TABLE 11

|  | Equation | subject | Values |
| :---: | :---: | :---: | :---: |
| 1 | $\mathrm{F}=\mathrm{ma}$ | m | $\mathrm{F}=4.00 \mathrm{kN} \quad \mathrm{a}=2.00 \mathrm{~ms}^{-2}$ |
| 2 | $\Delta \mathrm{Q}=\Delta \mathrm{U}+\Delta \mathrm{W}$ | $\Delta \mathrm{W}$ | $\Delta \mathrm{Q}=1.50 \mathrm{~mJ} \quad \Delta \mathrm{U}=750 \mu \mathrm{~J}$ |
| 3 | $\mathrm{s}=2 \pi \mathrm{r}$ | r | $\mathrm{s}=56.0 \mathrm{~cm}$ |
| 4 | $\mathrm{v}=\mathrm{u}+\mathrm{at}$ | u | $\mathrm{v}=10.0 \mathrm{~ms}^{-1} \quad \mathrm{a}=2.50 \mathrm{~ms}^{-2} \mathrm{t}=2.00 \mathrm{~s}$ |
| 5 | $\mathrm{v}=\mathrm{u}+\mathrm{at}$ | t | $\mathrm{v}=10.0 \mathrm{~ms}^{-1} \quad \mathrm{u}=0.00 \mathrm{~ms}^{-1} \quad \mathrm{a}=2.0 \mathrm{~ms}^{-2}$ |
| 6 | $\mathrm{I}_{\mathrm{o}}=\mathrm{I}_{\mathrm{rms}} \sqrt{ } 2$ | $\mathrm{I}_{\text {rms }}$ | $\mathrm{I}_{0}=8.00 \mu \mathrm{~A}$ |
| 7 | $s=u t+1 / 2 t^{2}$ | u | $\mathrm{s}=200 \mathrm{~m} \quad \mathrm{a}=5.00 \mathrm{~ms}^{-2} \mathrm{t}=8.00 \mathrm{~s}$ |
| 8 | $\mathrm{E}=1 / 2 \mathrm{mv}^{2}$ | v | $\mathrm{E}=12.0 \mathrm{MJ} \quad \mathrm{m}=6.00 \times 10^{4} \mathrm{~kg}$ |
| 9 | $\varepsilon=\frac{E}{Q}$ | Q | $\varepsilon=12.0 \mathrm{kV} \quad \mathrm{E}=360 \mathrm{~mJ}$ |
| 10 | $I=\frac{\Delta Q}{\Delta t}$ | $\Delta \mathrm{t}$ | $\mathrm{I}=50.0 \mu \mathrm{~A} \quad \Delta \mathrm{Q}=1.20 \mathrm{mC}$ |
| 11 | $\rho=\frac{m}{V}$ | m | $\rho=1000 \mathrm{kgm}^{-3} \mathrm{~V}=5.00 \times 10^{-6} \mathrm{~m}^{3}$ |
| 12 | $\rho=\frac{m}{V}$ | V | $\rho=13.6 \times 10^{3} \mathrm{kgm}^{-3} \mathrm{~m}=5.00 \mathrm{~kg}$ |
| 13 | $\mathrm{E}=1 / 2 \mathrm{Fe}$ | F | $\mathrm{E}=5.00 \mu \mathrm{~J} \quad \mathrm{e}=250 \mathrm{~nm}$ |
| 14 | $w=\frac{\lambda D}{s}$ | s | $\mathrm{w}=1.50 \mathrm{~mm} \quad \lambda=700 \mathrm{~nm} \quad \mathrm{D}=2.00 \mathrm{~m}$ |
| 15 | $\mathrm{hf}=\mathrm{E}_{1}-\mathrm{E}_{2}$ | $\mathrm{E}_{2}$ | $\mathrm{h}=6.63 \times 10^{-34} \mathrm{Js} \quad \mathrm{E}_{1}=2.18 \times 10^{-16} \mathrm{~J} \quad \mathrm{f}=2.46 \times 10^{15} \mathrm{~Hz}$ |
| 16 | $\mathrm{hf}=\Phi+\mathrm{E}_{\text {max }}$ | f | $\mathrm{h}=6.63 \times 10^{-34} \mathrm{Js} \quad \Phi=5.76 \times 10^{-19} \mathrm{~J} \quad \mathrm{E}_{\text {max }}=2.09 \times 10^{-19} \mathrm{~J}$ |
| 17 | $\mathrm{E}=1 / 2 \mathrm{mv}^{2}$ | m | $\mathrm{E}=16.0 \mathrm{GJ} \quad \mathrm{v}=2.32 \times 10^{3} \mathrm{~ms}^{-1}$ |
| 18 | $\varepsilon=\mathrm{IR}+\mathrm{Ir}$ | r | $\mathrm{R}=20.0 \mathrm{k} \Omega \quad \varepsilon=12.0 \mathrm{~V} \quad \mathrm{I}=0.500 \mathrm{~mA}$ |
| 19 | $\varepsilon=\mathrm{IR}+\mathrm{Ir}$ | I | $\varepsilon=6.00 \mathrm{~V} \quad \mathrm{R}=20.0 \Omega \quad \mathrm{r}=1.50 \Omega$ |
| 20 | $\mathrm{v}^{2}=\mathrm{u}^{2}+2 \mathrm{as}$ | a | $\mathrm{u}=1.00 \mathrm{~ms}^{-1} \quad \mathrm{v}=8.00 \mathrm{~ms}^{-1} \quad \mathrm{~s}=50.0 \mathrm{~m}$ |
| 21 | $\mathrm{s}=\mathrm{ut}+1 / 2 \mathrm{at}^{2}$ | a | $\mathrm{s}=100 \mathrm{~m} \quad \mathrm{u}=2.00 \mathrm{~ms}^{-1} \quad \mathrm{t}=6.00 \mathrm{~s}$ |

## Maths for Physics - Basic skills

## 11. ANSWERS

1. Expressing numbers in standard form.

## Table 1

|  | Value | What is it? | Expressed in <br> standard form |
| :---: | :---: | :---: | :---: |
| 1 | $9,460,528,400,000,000 \mathrm{~m}$ | a light year | $9.4605284 \times 10^{15}$ |
| 2 | $101,325 \mathrm{~Pa}$ | atmospheric <br> pressure | $1.01325 \times 10^{5}$ |
| 3 | $384,403,000 \mathrm{~m}$ | Moon-Earth <br> separation | $3.84403 \times 10^{8}$ |
| 4 | $299,790,000 \mathrm{~ms}^{-1}$ | speed of light | $2.9979 \times 10^{8}$ |
| 5 | $602,200,000,000,000,000,000,000$ <br> molecules per mole | Avogadro's <br> number | $6.022 \times 10^{23}$ |
| 6 | 0.02 m | microwave <br> wavelength | $2 \times 10^{-2}$ |
| 7 | 0.000000001 m | diameter of <br> an atom | $1 \times 10^{-9}$ |
| 8 | 0.000000515 m | wavelength <br> of visible <br> light | $5.15 \times 10^{-7}$ |
| 9 | $0.0000000172 \Omega \mathrm{~m}$ | resistivity of <br> copper | $1.72 \times 10^{-8}$ |

2. Expressing numbers to a specified number of significant figures

Table 2

| Number | Number of significant figures |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{1}$ |
| 3.14235 | 3.14 | 3.1 | 3 |
| 3.009442 | 3.01 | 3.0 | 3 |
| 45.336 | 45.3 | 45 | 50 |
| 72.49513 | 72.5 | 72 | 70 |
| 505.334 | 505 | 510 | 500 |
| 793.4591 | 793 | 790 | 800 |
| $3,045.778$ | 3050 | 3000 | 3000 |
| $5,139.113$ | 5140 | 5100 | 5000 |
| $76,493.2$ | 76,500 | 76,000 | 80,000 |
| $43,395.8$ | 43,400 | 43,000 | 40,000 |
| $3.9751 \times 10^{4}$ | $3.98 \times 10^{4}$ | $4.0 \times 10^{4}$ | $4 \times 10^{4}$ |
| $44.964 \times 10^{3}$ | $45.0 \times 10^{3}$ | $45 \times 10^{3}$ | $40 \times 10^{3}$ |

## Maths for Physics - Basic skills

Table 3

| Number | Number of significant figures |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{1}$ |
| 0.93552 | 0.936 | 0.94 | 0.9 |
| 0.95387 | 0.954 | 0.95 | 1 |
| 0.032532 | 0.0325 | 0.033 | 0.03 |
| 0.0403912 | 0.0404 | 0.040 | 0.04 |
| 0.0038461 | 0.00385 | 0.0038 | 0.004 |
| 0.00093571 | 0.000936 | 0.00094 | 0.0009 |
| $4.8835 \times 10^{-3}$ | $4.88 \times 10^{-3}$ | $4.9 \times 10^{-3}$ | $5 \times 10^{-3}$ |
| $34.9883 \times 10^{-5}$ | $35.0 \times 10^{-5}$ | $35 \times 10^{-5}$ | $30 \times 10^{-5}$ |

3. Calculating the percentage change in values

Table 4

| Initial value | Change | $\%$ | Final value |
| :---: | :---: | :---: | :---: |
| 2.55 | Increases by | 1.70 | 2.59 |
| 32.2 | Increases by | 28.4 | 41.3 |
| 892 | Decreases by | 35.2 | 578 |
| 90.3 | Increases by | 0.331 | 90.6 |
| 73.8 | Decreases by | 8.95 | 67.2 |
| 0.823 | Decreases by | 10.4 | 0.737 |
| 51.6 | Increases by | 3.29 | 53.3 |
| 274 | Decreases by | 6.93 | 255 |

4. Performing calculations on a calculator with numbers in standard form

Table 5

| Calculation | Answer |
| :--- | :---: |
| $3.11 \times 10^{4} \times 7.85 \times 10^{3}$ | $2.44 \times 10^{8}$ |
| $45.22 \times 10^{-3} \times 1.077 \times 10^{-6}$ | $4.87 \times 10^{-8}$ |
| $435 \times 10^{-9} \div 58.8 \times 10^{6}$ | $7.40 \times 10^{-15}$ |
| $0.55 \times 10^{5} \div 2.22 \times 10^{-5}$ | $2.48 \times 10^{9}$ |
| $\left(17.3 \times 10^{2}\right)^{2}$ | $2.99 \times 10^{6}$ |
| $3.33 \times 10^{-12} \times\left(4.053 \times 10^{6}\right)^{2}$ | $5.47 \times 10^{1}$ or 54.7 |
| $173 \times 10^{-9} \div\left(4.44 \times 10^{3}\right)^{3}$ | $1.98 \times 10^{-18}$ |

## Maths for Physics - Basic skills

## 5. Prefixes used in Physics

Table 6

| Prefix value | Base unit value |
| :---: | :---: |
| 1 GPa | $1 \times 10^{9} \mathrm{~Pa}$ |
| 1.75 kJ | $1.75 \times 10^{3} \mathrm{~J}$ |
| 0.35 MHz | $0.35 \times 10^{6} \mathrm{~Hz}$ |
| 32.0 mC | $32.0 \times 10^{-3} \mathrm{C}$ |
| 640 km | $640 \times 10^{3} \mathrm{~m}$ |
| 52 kg | 52 kg |
| 450 nm | $450 \times 10^{-9} \mathrm{~m}$ |
| $0.73 \mu \mathrm{~s}$ | $0.73 \times 10^{-6} \mathrm{~s}$ |
| 43.9 TV | $43.9 \times 10^{12} \mathrm{~V}$ |
| 470 pF | $470 \times 10^{-12} \mathrm{~F}$ |

Table 7

| Base unit value | Prefix value |
| :---: | :---: |
| $3.2 \times 10^{9} \mathrm{~m}$ | 3.2 Gm |
| $0.72 \times 10^{5} \mathrm{~N}$ | 72 kN |
| $1.27 \times 10^{4} \mathrm{~Hz}$ | 12.7 kHz |
| $3.2 \times 10^{-7} \mathrm{~s}$ | 320 ns or 0.32 ss |
| $2.35 \times 10^{-8} \mathrm{~m}$ | 23.5 nm |
| $48.3 \times 10^{7} \mathrm{~W}$ | 483 MW or 0.483 GW |
| $0.36 \times 10^{-11} \mathrm{~F}$ | 3.6 pF |
| $0.945 \times 10^{14} \mathrm{~V}$ | 94.5 TV |

## 6. Converting between units

Table 8

| Convert the following | Into this unit | Answer |
| :---: | :---: | :---: |
| $3.94 \mathrm{~mm}^{2}$ | $\mathrm{~m}^{2}$ | $3.94 \times 10^{-6}$ |
| 6.71 cm | m | $6.71 \times 10^{-2}$ |
| $0.883 \mu \mathrm{~m}^{2}$ | $\mathrm{~m}^{2}$ | $0.883 \times 10^{-12}$ |
| $78.3 \mathrm{~mm}^{3}$ | $\mathrm{~m}^{3}$ | $78.3 \times 10^{-9}$ |
| $68.5 \mathrm{~cm}^{2}$ | $\mathrm{~m}^{2}$ | $68.5 \times 10^{-4}$ |
| $3.81 \mu \mathrm{~m}$ | mm | $3.81 \times 10^{-3}$ |
| $28.8 \mathrm{~cm}^{3}$ | $\mathrm{~m}^{3}$ | $28.8 \times 10^{-6}$ |
| $7.31 \mu \mathrm{~m}^{2}$ | $\mathrm{~mm}^{2}$ | $7.31 \times 10^{-6}$ |

Maths for Physics - Basic skills
Table 9

| Angle in degrees | Angle in radians |
| :---: | :---: |
| 180 | 3.14 |
| 360 | 6.28 |
| 90.0 | 1.57 |
| 73.1 | 1.28 |
| 540 | 9.42 |
| 1.57 | 0.0274 |
| 18.0 | 0.314 |
| 180 | 3.14 |
| 270 | 4.71 |

## 7. Simple trigonometry

1) $x=3.35, \theta=38.9 \pm 0.1^{\circ}$;
2) $x=5.14, y=7.63$
3) $x=5.97, \theta=50.4 \pm 0.1^{0}$
4) $x=18.4, y=15.3 \pm 0.1$;
5) $x=18.5, y=16.3 \pm 0.1$;
6) $x=5.26, \theta=51.1 \pm 0.1^{0}$
8. Re-arranging formulae

Table 10
1.
$Q=\frac{E}{\varepsilon}$
8. $f=\frac{\phi+E_{m}}{h}$
2. $\Delta \mathrm{Q}=\mathrm{I} \Delta \mathrm{t}$
9. $v=\sqrt{\frac{2 E_{k}}{m}}$
3. $\mathrm{V}=\frac{\mathrm{m}}{\rho}$
10. $\Delta \mathrm{L}=\sqrt{\frac{2 \mathrm{E}}{\mathrm{k}}}$
4. $F=\frac{2 E}{\Delta L}$
11. $r=\frac{\varepsilon-\mathrm{IR}}{\mathrm{I}}$
5. $s=\frac{\lambda D}{w}$
12. $\mathrm{I}=\frac{\varepsilon}{\mathrm{R}+\mathrm{r}}$
6. $E_{2}=E_{1}-h f$
13. $\mathrm{a}=\frac{\mathrm{v}^{2}-\mathrm{u}^{2}}{2 \mathrm{~s}}$
7. $L=\frac{E A \Delta L}{f}$
14. $a=\frac{2(s-u t)}{t^{2}}$

## Maths for Physics - Basic skills

9. Performing complex calculations on a calculator
10. $1.229 \times 10^{7}$
11. $3.2 \times 10^{7}$
12. Answering a question (putting all the skills together)

Table 11

|  | Re-arranged equation | Numerical Answer |
| :---: | :---: | :---: |
| 1 | $\mathrm{m}=\mathrm{F} / \mathrm{a}$ | $\mathrm{m}=2.00 \times 10^{3} \mathrm{~kg}$ |
| 2 | $\Delta \mathrm{W}=\Delta \mathrm{Q}-\Delta \mathrm{U}$ | $\Delta \mathrm{W}=0.750 \times 10^{-3} \mathrm{~J}$ OR $\Delta \mathrm{W}=750 \times 10^{-6} \mathrm{~J}$ |
| 3 | $\mathrm{r}=\mathrm{s} / 2 \pi$ | $\mathrm{r}=8.91 \times 10^{-2} \mathrm{~m}$ |
| 4 | $\mathrm{u}=\mathrm{v}$-at | $\mathrm{u}=5.00 \mathrm{~ms}^{-1}$ |
| 5 | $\mathrm{t}=(\mathrm{v}-\mathrm{u}) / \mathrm{a}$ | $\mathrm{t}=5.00 \mathrm{~s}$ |
| 6 | $\mathrm{I}_{\mathrm{rms}}=\mathrm{I}_{0} / \sqrt{ } 2$ | $\mathrm{I}_{\text {rms }}=5.66 \times 10^{-6} \mathrm{~A}$ |
| 7 | $\mathrm{u}=\left(\mathrm{s}-1 / 2 \mathrm{at} \mathrm{t}^{2} / \mathrm{t}\right.$ | $\mathrm{u}=5.00 \mathrm{~ms}^{-1}$ |
| 8 | $\mathrm{v}=(2 \mathrm{E} / \mathrm{m})^{1 / 2}$ | $\mathrm{v}=20.0 \mathrm{~ms}^{-1}$ |
| 9 | $\mathrm{Q}=\mathrm{E} / \varepsilon$ | $\mathrm{Q}=3.00 \times 10^{-5} \mathrm{C}$ |
| 10 | $\Delta \mathrm{t}=\Delta \mathrm{Q} / \mathrm{I}$ | $\Delta \mathrm{t}=24.0 \mathrm{~s}$ |
| 11 | $\mathrm{m}=\rho \mathrm{V}$ | $\mathrm{m}=5.00 \times 10^{-3} \mathrm{~kg}$ |
| 12 | $\mathrm{V}=\mathrm{m} / \mathrm{\rho}$ | $\mathrm{V}=3.68 \times 10^{-4} \mathrm{~m}^{3}$ |
| 13 | $\mathrm{F}=2 \mathrm{E} / \mathrm{e}$ | $\mathrm{F}=40.0 \mathrm{~N}$ |
| 14 | $s=\lambda D / w$ | $\mathrm{s}=9.33 \times 10^{-4} \mathrm{~m}$ |
| 15 | $\mathrm{E}_{2}=\mathrm{E}_{1}-\mathrm{hf}$ | $\mathrm{E}_{2}=2.16 \times 10^{-16} \mathrm{~J}$ |
| 16 | $\mathrm{f}=\left(\Phi+\mathrm{E}_{\text {max }}\right) / \mathrm{h}$ | $\mathrm{f}=1.18 \times 10^{15} \mathrm{~Hz}$ |
| 17 | $\mathrm{m}=2 \mathrm{E} / \mathrm{v}^{2}$ | $\mathrm{m}=5950 \mathrm{~kg}$ OR $5.95 \times 10^{3} \mathrm{~kg}$ |
| 18 | $\mathrm{r}=(\varepsilon-\mathrm{IR}) / \mathrm{I}$ | $\mathrm{r}=4.00 \times 10^{3} \Omega$ |
| 19 | $\mathrm{I}=\varepsilon /(\mathrm{R}+\mathrm{r})$ | $\mathrm{I}=0.279 \mathrm{~A}$ |
| 20 | $\mathrm{a}=\left(\mathrm{v}^{2}-\mathrm{u}^{2}\right) / 2 \mathrm{~s}$ | $\mathrm{a}=0.630 \mathrm{~ms}^{-2}$ |
| 21 | $a=2(s-u t) / t^{2}$ | $\mathrm{a}=4.89 \mathrm{~ms}^{-2}$ |


[^0]:    $x=$ $\qquad$
    $\theta=$ $\qquad$

